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AUTOOSCILLATIONS IN DIELECTRIC SUSPENSIONS WITH THE "NEGATIVE" VISCOSITY EFFECT

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We consider an active system, consisting of dielectric particles, under the action of electric field. Rotation of particles in liquid is synchronized by the Couetta flow. To observe oscillations, we put a plate on the liquid layer and attach it to a spring. A situation is considered when the polarization time is small in comparison with the typical mechanical relaxation time. In such case, the stationary point can be unstable and, due to the "negative" viscosity effect, autooscillations of the plate could be observed, if the strength of an external electric field is beyond some critical value. We derive the exact range of parameters, when autooscillations of the plate are possible, and numerically calculate the period. It is also shown that beyond this range the system becomes stable and stressed, thus imitating the behaviour of the muscle cells.

Introduction. Active systems have attracted much interest recently. They are responsible for the functioning of the cell, motility of different microorganisms [1]. Different unusual properties of these systems are known, for example, the muscle cells of oysters develop high tension keeping them closed by expenditure of energy. Creation of different systems (gels and other), exhibiting the properties of active systems, has been started in different labs recently [2]. Here we are exploring the properties of an active system – a suspension of dielectric particles in a liquid of low conductivity, in which external energy is supplied by an electric field [3]. It should be pointed out that electrostatic rotary machines are used by bacteria to sustain their motility [1].

The dielectric suspension is known to have some properties typical of the active living systems – the possibility to sustain the stretched quiescent state, the autooscillations, which are observed for insect muscles [4], and others. The physical system considered consists of a dielectric suspension with internal rotations, which models, for example, the action of the molecular motors between two plates, one of which is free to move and connected to the spring. This mimics the thin filaments of the muscle cells, where titin and nebulin rulers serve as elastic springs in the sarcomere [1]. We have shown that depending on the physical parameters, the system exhibits different regimes of autooscillations. In some range of the parameters the stressed steady state sustained by the internal rotations is unstable and autooscillations arise, imitating in such a way the behaviour of muscle cells.

The polarization relaxation equation is given by

$$\frac{\mathrm{d}\mathbf{P}}{\mathrm{d}t} = [\mathbf{\Omega} \times \mathbf{P}] - \frac{1}{\tau}(\mathbf{P} - \chi\mathbf{E}), \tag{1}$$

where $\mathbf{\Omega}$ is the angular velocity of a rotating particle, τ is the Maxwell relaxation time and $\chi = \chi_0 - \chi_\infty$, where χ_0 and χ_∞ denote susceptibilities of the suspension polarization at low and high electric field frequencies, correspondingly. Neglecting the inertia of a small rotating particle, the balance of viscous and electrical torques gives

$$\alpha(\mathbf{\Omega} - \mathbf{\Omega_0}) = [\mathbf{P} \times \mathbf{E}], \tag{2}$$

where $\mathbf{\Omega_0}$ is the vorticity of a macroscopic flow and α is the rotational friction coefficient of the particles per unit volume. Neglecting the inertia of the free plate, the force balance on the plate along the x-axis reads

$$-\eta\frac{S}{h}\frac{\mathrm{d}x}{\mathrm{d}t} - \frac{S}{2}[\mathbf{P}\times\mathbf{E}]\cdot\mathbf{e}_z - kx = 0, \tag{3}$$

where k is a spring constant, η is the viscosity of the liquid, S is the area of the plate and h is the thickness of the liquid layer. The flow vorticity, $\mathbf{\Omega_0}$, in the Couetta flow assumption can be expressed as

$$\mathbf{\Omega_0} = -\frac{1}{2h}\frac{\mathrm{d}x}{\mathrm{d}t}\mathbf{e}_z. \tag{4}$$

From $\mathbf{\Omega} = \Omega\mathbf{e}_z$, $\mathbf{E} = E\mathbf{e}_y$ and $\mathbf{P} = P_x\mathbf{e}_x + P_y\mathbf{e}_y$ we get $[\mathbf{P}\times\mathbf{E}] = EP_x\mathbf{e}_z$ and $[\mathbf{\Omega}\times\mathbf{P}] = -\Omega P_y\mathbf{e}_x + \Omega P_x\mathbf{e}_y$, thus by excluding Ω from (1, 2, 3, 4), we obtain a set of equations

$$\begin{cases} \dfrac{\eta S}{kh}\dfrac{\mathrm{d}x}{\mathrm{d}t} = -\dfrac{SE}{2k}P_x - x \\ \dfrac{\mathrm{d}P_x}{\mathrm{d}t} = \dfrac{1}{2h}\dfrac{\mathrm{d}x}{\mathrm{d}t}P_y - \dfrac{E}{\alpha}P_xP_y - \dfrac{1}{\tau}P_x \\ \dfrac{\mathrm{d}P_y}{\mathrm{d}t} = \dfrac{E}{\alpha}P_x^2 - \dfrac{1}{\tau}P_y - \dfrac{1}{2h}\dfrac{\mathrm{d}x}{\mathrm{d}t}P_x + \dfrac{\chi}{\tau}E. \end{cases} \tag{5}$$

For a spontaneous rotation of particles to take place, the strength of the external electric field must satisfy the condition $E > E_c$, where $E_c^2 = -\alpha/\chi\tau$ (of course $\chi < 0$ is necessary). The characteristic relaxation time of the plate is $\tau_p = \eta S/kh$. By substituting $t = \tau_p \mathrm{t}$ in (5), the plate relaxation time τ_p is introduced as a time scale. Similarly, by substituting $x = 2\mathrm{x}h\tau_p/\tau$ and $P_i = \chi E\mathrm{P}_i$, we obtain the following dimensionless set of differential equations:

$$\begin{cases} \dfrac{\mathrm{dx}}{\mathrm{dt}} = -\mathrm{x} + ae\mathrm{P}_x \\ \dfrac{\tau}{\tau_p}\dfrac{\mathrm{dP}_x}{\mathrm{dt}} = \dfrac{\mathrm{dx}}{\mathrm{dt}}\mathrm{P}_y + e\mathrm{P}_x\mathrm{P}_y - \mathrm{P}_x \\ \dfrac{\tau}{\tau_p}\dfrac{\mathrm{dP}_y}{\mathrm{dt}} = -\dfrac{\mathrm{dx}}{\mathrm{dt}}\mathrm{P}_x - e\mathrm{P}_x^2 - \mathrm{P}_y + 1, \end{cases} \tag{6}$$

where the parameters e and a are expressed as follows: $e = E^2/E_c^2$ and $a = \alpha/4\eta$.

Autooscillations. Let us examine the case, when the Maxwell relaxation time τ for a particle is much smaller than the typical plate relaxation time τ_p, i.e., $\tau/\tau_p \to 0$. Thus, from (6) we obtain an algebraic set of equations

$$\begin{cases} \mathrm{v} = -\mathrm{x} + ae\mathrm{P}_x \\ \mathrm{vP}_y + e\mathrm{P}_x\mathrm{P}_y - \mathrm{P}_x = 0 \\ \mathrm{vP}_x + e\mathrm{P}_x^2 + \mathrm{P}_y = 1, \end{cases} \tag{7}$$

where $\mathrm{v} = d\mathrm{x}/d\mathrm{t}$. By excluding v from (7), one can find that the components of particle's polarization vector satisfy $\mathrm{P}_x^2 + \mathrm{P}_y^2 = \mathrm{P}_y$.

Exclusion of P_x and P_y from (7) gives a force (x) and velocity (v) relationship for the active system:

$$(\mathrm{v}+\mathrm{x})^3 + 2a\mathrm{v}(\mathrm{v}+\mathrm{x})^2 + a^2(\mathrm{v}+\mathrm{x})(1 - e + \mathrm{v}^2) - a^3e\mathrm{v} = 0. \tag{8}$$

Let us denote the left side of (8) with $F(\mathrm{x}, \mathrm{v})$. Independently of the values of e and a, there always exists a trivial solution $x = 0$, $v = 0$ for the equation $F(\mathrm{x}, \mathrm{v}) = 0$. The exact shape of the curve depends on the both parameters, and, in overall for the given a, four cases are possible there. They are shown in Fig.1 in an ascending order of e.

The number of cross-points of the implicit function (8) with the abscissa axis is related to the number of roots of $F(0, \mathrm{v}) = 0$. If $e > 1/(a+1)$, there are two nontrivial roots $\mathrm{v}_{1,2} = \pm a\sqrt{e(a+1) - 1}/(a+1)$ and, therefore, three cross-points of the abscissa axis, as shown in Fig.1(b,c,d). If $e \leq 1/(a+1)$, there is only one cross-point (Fig.1a).

Similarly, $F(\mathrm{x}, 0) = 0$ also has two nontrivial roots $\mathrm{x}_{1,2} = \pm a\sqrt{e-1}$, thus the plot of (8) crosses the ordinate axis three times, when $e > 1$, as shown in Fig.1(c,d). These cross-points are stationary points and appear due to a force balance created by the spontaneous rotation of active particles. If $e \leq 1$, there is only one stationary point, see Fig.1(a,b).

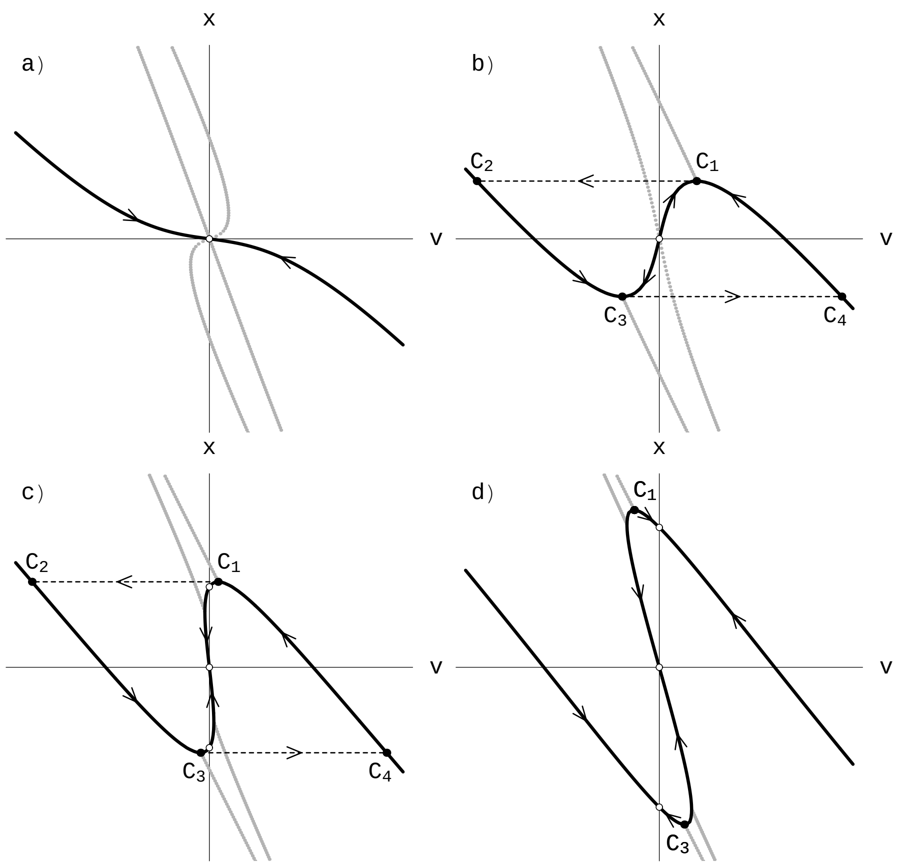

Fig. 1. Phase portraits of (8), where $a = 1.3$ and a) $e = 0.4$; b) $e = 0.8$; c) $e = 1.2$; d) $e = 2.0$. The black curve is an implicit plot of (8) – it contains all real roots of equation (8) and, therefore, shows all possible states of the system. Gray curves show real parts of roots with one complex component. White points are stationary (where $v = 0$). Arrows indicate the direction of motion. Dashed lines indicate jumps, where the velocity changes momentarily. In cases b) and c) a cycle is formed.

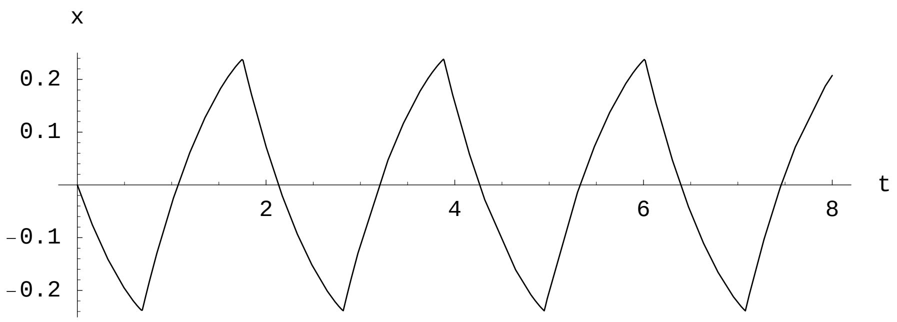

Fig. 2. Shape of oscillations x(t), where $a = 1.3$ and $e = 0.8$.

To determine the stability of stationary points, one should notice that the positive slope of x(v) in Fig.1 corresponds to the negative friction coefficient of the active system [5]. In case a), the only stationary point is stable. In case b), it becomes unstable. In case c), it becomes stable again, but two new unstable stationary points appear. In case d), all three stationary points become stable. It is interesting to note that there are states, where the differential friction coefficient in some range of the parameters explored below is negative. This causes the autooscilations around the stressed states.

As discussed above, in cases b) and c), there are unstable stationary points. In these cases at point C_1 in Fig.1 the coordinate x and velocity v are positive, but further the increase of x is not possible and a jump to C_2 happens, where x is the same, but the velocity of the oscillating plane is opposite. Similarly, a jump from C_3 to C_4 happens and a closed cycle C_1, C_2, C_3, C_4 is formed up. The typical shape of autooscillations is shown in Fig.2.

Now let us find all values of the parameters e and a, for which such periodic behaviour can be observed. For the jump from C_1 to C_2 to happen, it is necessary that the maximum of function x(v), i.e., point C_1, lies in the first quadrant, otherwise there will be a stable stationary point on the x-axis, which cannot be crossed, see Fig.1d. Thus, we have to solve $\partial F\big(\mathrm{x}(\mathrm{v}), \mathrm{v}\big)/\partial \mathrm{v} = 0$ together with $F\big(\mathrm{x}(\mathrm{v}), \mathrm{v}\big) = 0$ with restrictions $\mathrm{x} > 0$ and $\mathrm{v} > 0$. This leads to inequalities

$$\frac{1}{a+1} < e < 2\frac{a+1}{a+2}, \tag{9}$$

where $a > 0$. This is summarized in Fig.3. One can see that the periodic behaviour cannot be observed, if e is too small, i.e., $e \leq 1/(a+1)$ - case a), or e is too big, i.e., $e \geq 2(a+1)/(a+2)$ - case d). In other cases autooscillations can be observed in Fig.1(b,c) where dashed lines indicate the jumps.

To define the period of autooscillations, we consider the symmetry of the implicit function plot $F(\mathrm{x}, \mathrm{v}) = 0$ and that the jump from C_1 to C_2 happens momentarily (of course, inequalities (9) must be satisfied for the jump to happen). By perceiving v as a function of x in the segment C_4C_1, one can find the period by integrating $d\mathrm{x}/\mathrm{v}(\mathrm{x})$, i.e.,

$$T = 2\int_{-\mathrm{x}_c}^{+\mathrm{x}_c} \frac{d\mathrm{x}}{\mathrm{v}(\mathrm{x})}$$

where x_c is the ordinate of critical point C_1. Period dependence on the parameters is illustrated in Fig. 4.

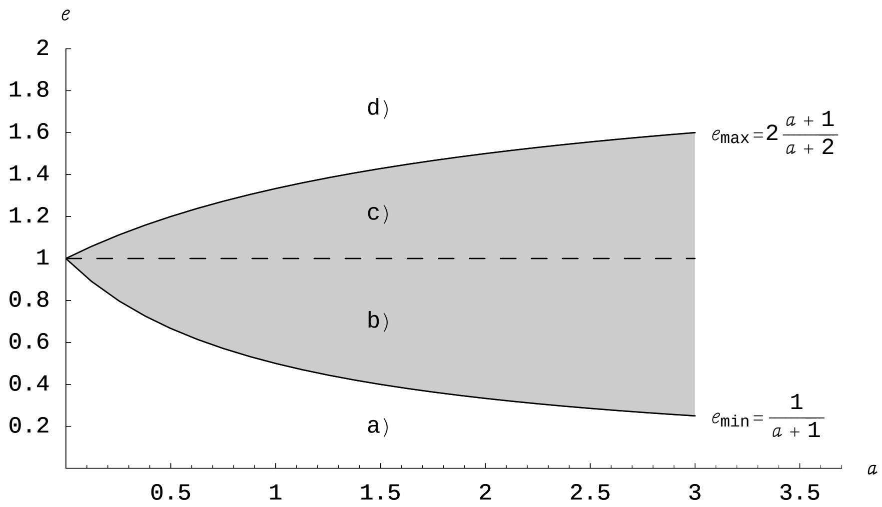

Fig. 3. Summary of all possible cases shown in Fig.1. The gray region indicates the values of the parameters a and e, when autooscillations take place.

Dielectric suspension as an active system. It presumably makes sense to look on the results considered here in a broader context of the natural active systems, for example, such as muscles [6]. The dependence of the shortening velocity of the muscle on the applied force f is well described by the Hill equation [6]

$$v = \frac{B(f_0 - f)}{A + f} \tag{10}$$

This equation arises naturally in the frame of the sliding filament model. Let the total number of the cross-bridges between actin and myosin filaments be n_0, with the number of active sites for the given load being n. Let the force produced by each active site be f_0. Then the force balance reads

$$nf_0 - n_0 f - \beta nv = 0 \tag{11}$$

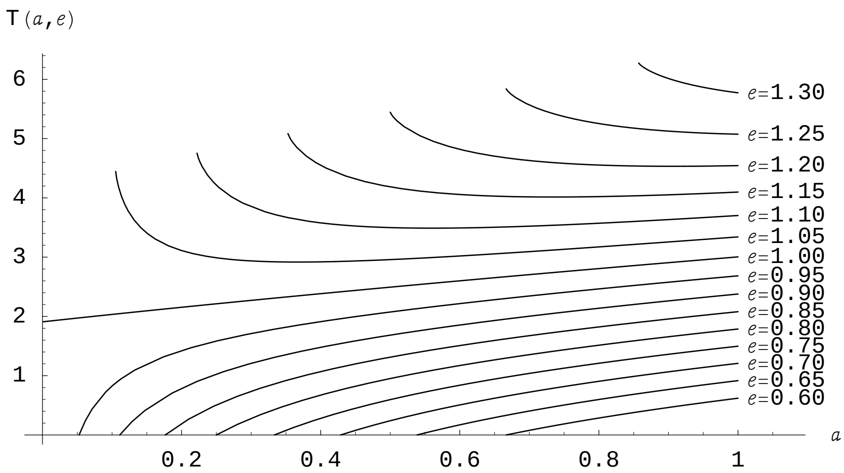

Fig. 4. Period dependence on a.

where β is the friction coefficient per active site, and f being a load per site. Assuming the dependence of the fraction of active sites on the load (r is the fraction of active sites in the absence of the load)

$$\frac{n}{n_0} = r + (1-r)\frac{f}{f_0},$$

the Hill equation is obtained, where the constants A, B are expressed as follows $A = rf_0/(1-r)$ and $B = rf_0/\beta(1-r)$. According to the Hill equation, the maximum shortening velocity v_{max} of the muscle cell at $f = 0$ is Bf_0/A, but the maximum force developed by the muscle under isometric conditions is f_0. The corresponding values for our active system, according to Eq. (8), are $v_{max} = a\sqrt{e(a+1)-1}/(a+1)$ and $f_0 = a\sqrt{e-1}$. This allows one to consider a force-velocity dependence given by the Hill equation for the present active system. It reads

$$f = \frac{Bf_0 - Av}{v+B}$$

which, being rewritten in terms of the parameters of dielectric suspension, reads

$$f = \frac{\sqrt{e-1}}{\sqrt{(a+1)e-1}} \frac{a\sqrt{(a+1)e-1} - v(a+1)}{1+v/B} \tag{12}$$

From the force-velocity dependence given by the solution of Eq. (8) it is possible to define the constant B^{-1} by fit with function (12), which in dependence on the parameter e is shown in Fig.5 (at $a = 0.2$) We see that it is negative and thus gives the evidence that the behaviour of the present active system is more complicated, as described by the Hill equation. Nevertheless, there is a way to generalize the simple derivation of the Hill equation, which allows one to qualitatively explain the peculiarities of the present system. Since the negative viscosity effect is typical of it, it is natural to assume that the total friction force has two counterparts – one negative due to active sites and other positive due to the overall friction. In this case, equation (11) reads

$$nf_0 - n_0 f + \beta_0 nv - \beta n_0 v = 0. \tag{13}$$

Here β_0 describes the negative friction of active sites. Assuming the same dependence of the active site friction on the load as above, the force-velocity relationship reads

$$f = \frac{A(v_{max} - v)}{B - v}$$

where $A = f_0(\beta - \beta_0 r)/\beta_0(1-r)$; $B = rf_0/\beta_0(1-r)$ and $v_{max} = rf_0/(\beta - \beta_0 r)$. Note that the negative value of the constant B^{-1} quite naturally can be associated with the negative friction in a system, which, in our case, arises due to the rotations of dielectric particles. Since the increasing of B^{-1} is related to transition to a non-monotonous force-velocity dependence, as illustrated in Fig.1c, when spontaneous oscillations of the quiescent state arise, it seems justified to conclude that the spontaneous oscillations observed for muscles of some insects [1] could be determined by the negative friction coefficient in the system of sliding myosin-actin filaments. Models based on this property of the systems of molecular motors, which predict the spontaneous oscillations of the muscle cells, have already appeared [4].

Conclusions. We have illustrated here that the dielectric suspension in the electric field behaves like an active system of the living world. The obtained results can be applied for creating artificial active systems for microfluidics, and others.

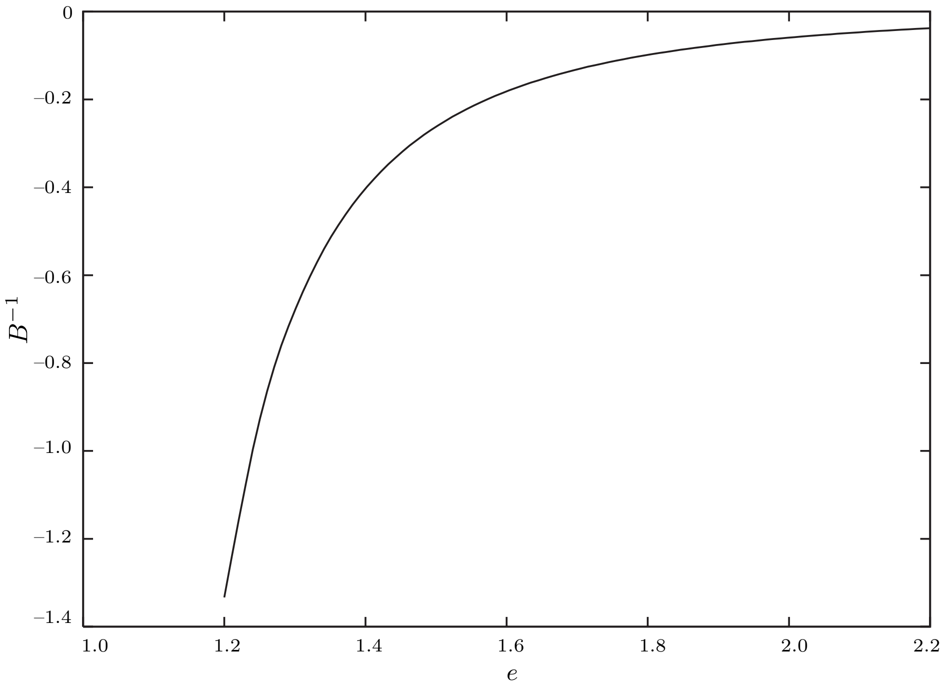

Fig. 5. Dependence of the Hill equation parameter on e.

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